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原　著

A Dynamical Model of a Geyser Induced by Gas Inflow

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間欠泡沸泉の動力学モデル

鏡　裕行[1)]

要　旨

室内モデル実験の詳細な観察に基づいた，間欠泡沸泉の動力学モデルを構築した．この動力学モデルの数値シミュレーションにより，噴出動力学の様々な地下パラメタ（地下空洞体積，水柱の長さ（高さ）など）依存性を明らかにした．また，噴出管壁と水の間の摩擦を考慮した動力学モデルに改良した．この改良された動力学モデルの数値シミュレーションにより，噴出管壁と水の間の摩擦によって噴出高さが減衰することが明らかとなった．

キーワード：間欠泡沸泉，動力学モデル，数値シミュレーション，摩擦の効果

Abstract

I constructed a basic dynamical model of a geyser induced by gas inflow through detailed observation of the indoor model experiments. And I clarified dependence of spouting dynamics on various underground parameters (volume of the underground space, length (height) of a water pole and so on) through numerical simulation of the dynamical model. And I improved the dynamical model, that is, I took effects of friction between the walls of the spouting pipe and water into account. And I clarified how spouting height was damped by friction between the walls of the spouting pipe and water through numerical simulation of the improved dynamical model.

Key words : geyser induced by gas inflow, dynamical model, numerical simulation, effects of friction

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1. Introduction

A geyser is defined as a natural spring that sends hot water and steam intermittently into the air from a hole in the ground. Geysers are classified into two types dependent on inducer. Namely, one is a geyser induced by boiling and the other is a geyser induced by gas inflow. A geyser induced by gas inflow spouts due to pressure of underground gas at the temperature under the boiling point of water. And only a few studies about its mechanism have been proposed. Iwasaki (1944, 1962) constructed experimentally some geyser models of cold waters and gases with cavities and estimated spouting time and pause time using the gas supply rate as a parameter based on the simple calculation of gas balance. But the discussion is too simple to estimate the spouting or pause time dependent on various underground parameters and does not discuss spouting dynamics of a geyser induced by gas inflow.

So I derived a static (mathematical) model and a dynamical model of a geyser induced by gas inflow based on detailed observation of the indoor model experiments and have modified the dynamical model in diverse ways (Kagami, 2006 ; Kagami, 2007 ; Kagami, 2009 ; Kagami, 2010a ; Kagami, 2010b ; Kagami, 2010c ; Kagami, 2011 ; Kagami, 2012 ; Kagami, 2015a ; Kagami, 2015b). But I have not discuss results of numerical simulation of the dynamical model in detail in any papers yet, though the expansion and application of the dynamical model were reported in many papers as mentioned above. So in this paper we discuss the results of numerical simulation of the dynamical model in detail for the sake of understanding characteristics of the dynamical model.

2. Model

2.1 A basic dynamical model of a geyser induced by gas inflow

From the results of the indoor model experiments of the geyser induced by gas inflow, I understood that a beginning of spouting is made by the loss of surface tension supporting a lump of water packed in a pipe leading to a spouting exit. Namely, in the model experiments the underground situation shown in Fig. 1 is assumed. A spouting hole is deep and leads to a space where gas and water are supplied at a constant rate at the deep position under the ground. Before a beginning of spouting pressure of gas in the space is supported by surface tension on the lower interface between water and gas (and gravity acting on the mass of a lump of water packed in the hole (pipe) and the pressure of the atmosphere). But when a value of pressure of gas in the space becomes larger than a threshold, the surface tension comes not to be able to support pressure of gas in the space. Then a lump of water packed in a pipe leading to a spouting exit begins to move up to the exit on the ground. In a basic dynamical model of a geyser induced by the inflow of gas, the dynamics of a lump of water packed in the pipe is discussed.

When the pressure of gas in the space just before a lump of water's beginning to move up to the exit on the ground is put as p_i, p_i is represented as ;

$$p_i = p_0 + \rho g H + f_k \tag{1}$$

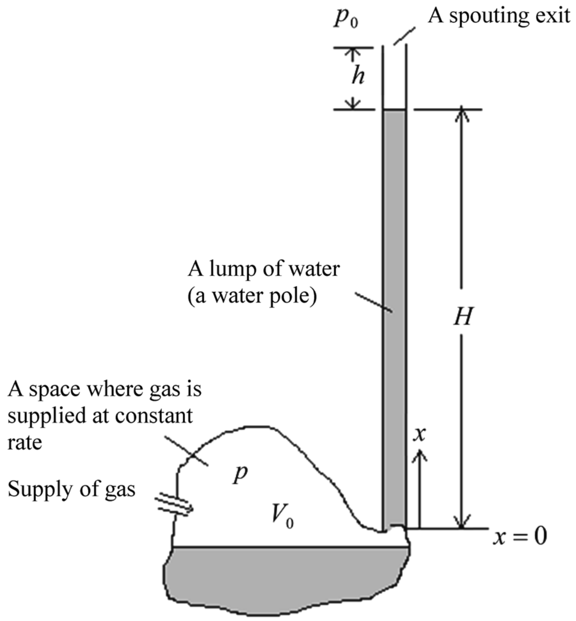

Fig. 1　An illustration of a geyser induced by inflow of gas.

where p_0 represents the pressure of the atmosphere, ρ represents density of water, g represents gravity acceleration, H represents length of a lump of water packed in the pipe from the lower interface between water and gas to the upper one and f_k represents pressure due to surface tension on the lower interface between water and gas. And f_k is represented as ;

$$f_k = \frac{2\sqrt{\pi}\gamma\cos\alpha}{\sqrt{S}} \tag{2}$$

where γ represents a coefficient of surface tension, α represents contact angle and S represents an area of a cross section of the pipe filling a lump of water.

When a lump of water packed in the hole begins to move up, f_k is regarded as $f_k \to 0$. Then when an upper direction of a vertical line is regarded as a plus direction of x-axis, an equation of motion of the lump of water is written as ;

$$\rho SH\frac{d^2x}{dt^2} = pS - \rho gSH - p_0S \tag{3}$$

where p represents the pressure of gas in the underground space. Here, x is regarded as a position of the lower interface between water and gas of the water pole and friction between the walls of the pipe and water is ignored.

When it is assumed that gas in the underground space is ideal gas and changes isothermally,

$$d\left(\frac{pV}{n}\right) = 0 \tag{4}$$

where V represents volume of gas filled in the underground space and n represents molar number of it is realized.

From equation (4),

$$npdV + nVdp - pVdn = 0 \tag{5}$$

is derived.

When it is assumed that $x = 0$ and $V = V_0$ just before the lump of water begins to move up, we can write V as ;

$$V = V_0 + Sx \tag{6}$$

From equation (6),

$$dV = Sdx \tag{7}$$

is derived.

From the assumption that gas is supplied at a constant rate in the underground space,

$$\frac{dn}{dt} = \beta \tag{8}$$

where β is constant is derived. From equation (8), n can be represented as;

$$n = n_0 + \beta t \tag{9}$$

where n_0 represents molar number when $p = p_i$ and $V = V_0$. On this account we can write using equation (1) as ;

$$n_0 = \frac{p_i V_0}{RT} = \frac{V_0}{RT}\left(p_0 + \rho g H + f_k\right) \tag{10}$$

Applying equation (6) - (9) to equation (5),

$$\left(n_0 + \beta t\right) pS \frac{dx}{dt} + \left(n_0 + \beta t\right)\left(V_0 + Sx\right)\frac{dp}{dt} = \left(V_0 + Sx\right) p\beta \tag{11}$$

is derived. And from equation (3),

$$\frac{dp}{dt} = \rho H \frac{d^3 x}{dt^3} \tag{12}$$

is derived. From equation (11) and (12) we can get

$$\left(n_0 + \beta t\right)\left(V_0 + Sx\right)\rho H \frac{d^3 x}{dt^3} + \left(n_0 + \beta t\right) pS \frac{dx}{dt} = \left(V_0 + Sx\right) p\beta \tag{13}$$

x, that is, a position of the lower interface between water and gas of the water pole moves obeying equation (13).

In general, the effects of surface tension are smaller with decreasing space size. Therefore, in this model, it is assumed that the actual gate connecting the spouting pipe and the underground space is enough small and doesn't resemble the expansion of the shape shown in

Fig. 1. It is thought that large volume of the underground space consists of the sum of small volume of the underground small caves which are connected each other by a network. This assumption is indirectly supported through video observations inside the conduits of erupting geysers by Belousov *et al.* (2014).

2. 2 An improved dynamical model in which effects of friction working between the wall's surface along the edge of the paths of water and water are taken into account

In the improved dynamical model, I take the effects of friction between the walls of the spouting pipe and water into account. The concrete methods of introduction of the effects of friction are explained in the following.

Taking effects of friction between the walls of the spouting pipe and water into account means that we regard a water pole packed in the spouting pipe as viscous fluid. If we formally assume the water pole as viscous fluid, we have to consider friction between water and water in the water pole. That means considering dynamics of viscous fluid and a subject will become much complicated.

So I consider pseudo-friction effects in which only friction between the walls of the spouting pipe and water is taken into account. Actually, because viscous fluid flowing in a circular pipe obeys Poiseuille's law which argues friction between the walls of the spouting pipe and water is largest in the circular pipe, the pseudo-friction effects are not beside the mark very much.

In the beginning, in numerical experiments solving an equation of motion of the water pole clarify its velocity at a moment. From the velocity and an area of cross section of the pipe S, flux of fluid V_f is derived.

On the other hand, from Poiseuille's law distribution of velocity u, of fluid is represented as :

$$u = B\left(a^2 - r^2\right) \tag{14}$$

where a represents a radius of the spouting pipe, r represents length from the center of a cross section of the spouting pipe to the direction perpendicular to the wall of the pipe and B is coefficient. Calculating flux V_B of fluid from equation (14),

$$V_B = \int_0^a 2\pi r u dr = \frac{\pi B a^4}{2} \tag{15}$$

is derived. From equation (15),

$$B = \frac{2V_B}{\pi a^4} \tag{16}$$

is derived.

While, from equation (14),

$$\frac{du}{dr} = -2Br \tag{17}$$

is derived. Therefore we can get

$$\frac{du}{dr}\Big|_{r=a} = -2Ba \tag{18}$$

On the other hand, inner friction force f is written as :

$$f = \eta A \frac{du}{dr} \tag{19}$$

where η represents viscosity coefficient and A represents an area where water keeps in touch with a wall of a pipe or water.

From

$$V_f = V_B \tag{20}$$

and equation (16), (18) and (19), we can write friction force f_w between the wall of the pipe and water as :

$$f_w = \frac{8\eta\pi H V_f}{S} \tag{21}$$

Furthermore, a direction of friction force f_w is opposite to that of velocity of the water pole $\frac{dx}{dt}$.

Then a term of friction force f_w is added to an equation of motion of the lump of the water (equation (3)). Hereafter the same discussion as that in the basic dynamical model is developed.

Concretely, we can write

$$V_f = S\frac{dx}{dt} \tag{22}$$

Noticing a sign of f_w, from equation (3), (21) and (22) we get

$$\begin{aligned}\rho SH\frac{d^2x}{dt^2} &= pS - \rho gSH - p_0S - f_w \\ &= pS - \rho gSH - p_0S - 8\pi\eta H\frac{dx}{dt}\end{aligned} \tag{23}$$

From equation (23), we get

$$\rho SH\frac{d^3x}{dt^3} = S\frac{dp}{dt} - 8\pi\eta H\frac{d^2x}{dt^2} \tag{24}$$

Finally, from equation (11) and (24) we get

$$(n_0 + \beta t)(V_0 + Sx)\rho H\frac{d^3x}{dt^3} + \frac{8\pi\eta H}{S}(n_0 + \beta t)(V_0 + Sx)\frac{d^2x}{dt^2} + (n_0 + \beta t)pS\frac{dx}{dt} = (V_0 + Sx)p\beta \tag{25}$$

Second term of equation (25) is a newly added term that represents effects of friction between the walls of the pipe and water.

3.　Results and Discussion

3.1　Results of numerical simulation of the basic dynamical model and discussion

I show some results of numerical simulation of the basic dynamical model of a geyser induced by gas inflow as follows.

In the beginning, I show the dependence of variation of height of a water pole (x) on length (height) of a water pole (H) during spouting in Fig. 2. Adopted values of parameters are shown in Table 1. The values are decided based on expected values. f_k is calculated using equation (2) in case of $\alpha = 30°$ and $S = 1$ [m^2]. In this case, S represents not an area of a cross section of the pipe filling a lump of water in the experimental system but an expected area of a cross section of a pipe connected just to the real underground space. On the other hand, the value of S in Table 1 represents an expected value (close to the observed value) of a cross section of the spouting pipe. And the value of V_0 represents the expected sum of small volume of the underground small caves which are connected each other by a network, as described above. And the value of β is estimated based on the total volume of spouted water during a spouting mode at a real geyser induced by the inflow of gas. While spouted water is removed from the water pole during spouting, the water is not removed from the water pole before spouting. From Fig. 2, we see that the higher a water pole (H) is, the smaller an amplitude of the water pole's oscillation is and the longer a spouting period (a period of the water pole's oscillation) is.

Table 1　Adopted values of parameters in numerical simulations.

f_k	2.24×10^{-1} [N/m^2]
ρ	1.0×10^{3} [kg/m^3]
S	1.0×10^{-2} [m^2]
p_0	1.01×10^{5} [N/m^2]
g	9.8×10^{0} [kg· m/s^2]
V_0	6.0×10^{4} [m^3]
β	1.0×10^{-3} [mol/s]
R	8.31×10^{0} [N· m/K/mol]
T	3.20×10^{2} [K]

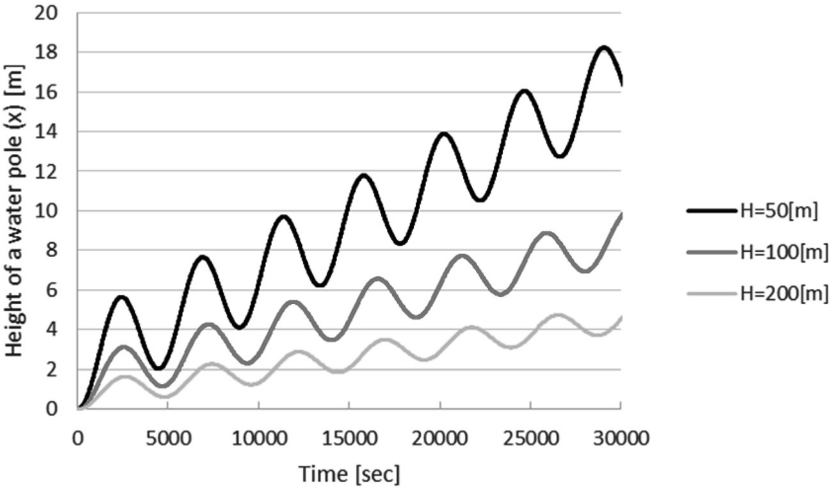

Fig. 2　Dependence of variation of height of a water pole (x) on length (height) of a water pole (H) during spouting.

Then I show the dependence of variation of height of a water pole (x) on length (height) of a water pole (H) before spouting in Fig. 3. Adopted values of parameters are the same as ones sown in Table 1. The characteristics seen from Fig. 3 resemble ones seen from Fig. 2. But the characteristics are a little different from ones seen from Fig. 2 because there is a loss of a water pole due to water's spouting in case of the former.

The difference is shown in Fig. 4. h in a legend of Fig. 3. 4 means a length (height) between a spouting exit and the upper surface of a water pole at the beginning. Though in case of $h=30$ the spouting has not started yet in the figure, in case of $h=30$ the spouting has already started. And the length (height) of water poles (H) before spouting is same in both cases. From Fig. 4,

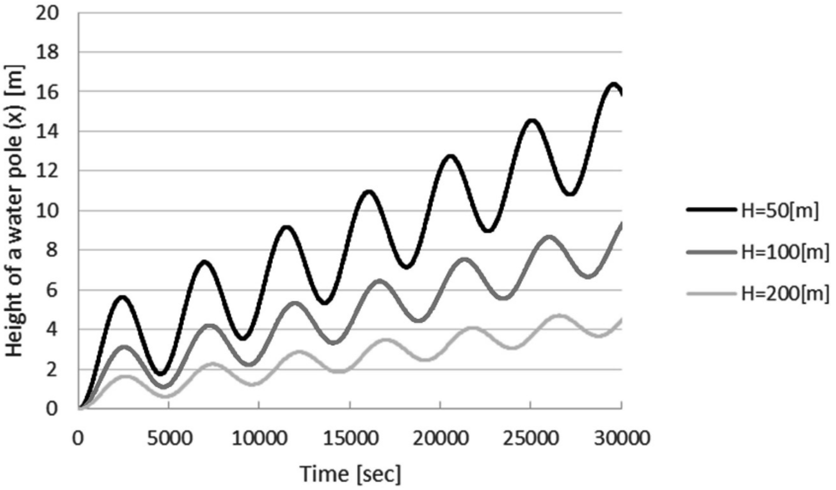

Fig. 3　Dependence of variation of height of a water pole (x) on length (height) of a water pole (H) before spouting.

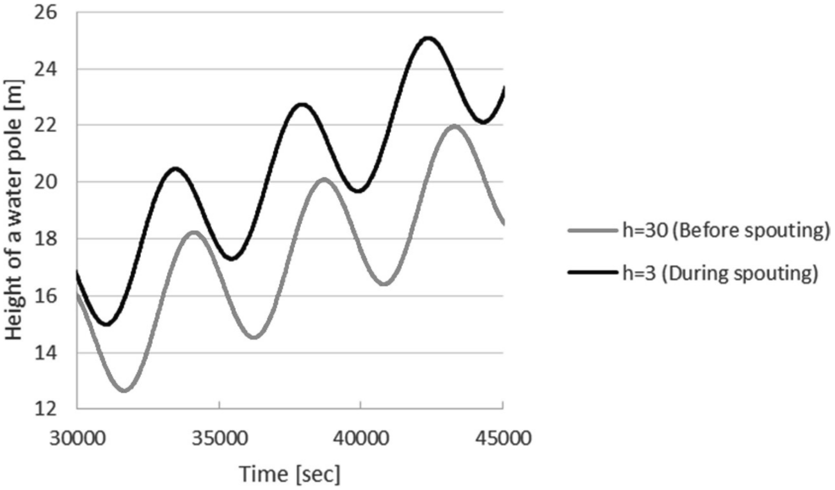

Fig. 4　Difference of variation of height of a water pole (x) between before spouting and during spouting.

we see that spouting period becomes shorter after spouting began. For substantial length (height) of water poles (*H*) becomes shorter after spouting begins.

Next, I show the dependence of variation of height of a water pole (*x*) on pressure due to surface tension on the lower interface between water and gas (f_k) in Fig. 5. Adopted values of parameters are the same as ones shown in Table 1 except for the value of f_k and $H=100$ [m]. From Fig. 5, We see that the larger pressure due to surface tension on the lower interface between water and gas (f_k) is, the larger an amplitude of the water pole's oscillation is. For f_k represents strength to push up a water pole. On the other hand, spouting period does not depend on f_k because f_k has an effect on only strength pushing up a water pole.

Next, I show the dependence of variation of height of a water pole (*x*) on volume of underground space (V_0) in Fig. 6. Adopted values of parameters are the same as ones sown in Table 1 except for the value of V_0 and $H=100$ [m]. Incidentally it may be thought that f_k is pressure due to not only above-mentioned surface tension but also other power. From Fig. 6, we see that the larger volume of underground space (V_0) is, the larger an amplitude of the water pole's oscillation is and the longer a spouting period is. Namely, the volume of underground space (V_0) affects both an amplitude of the water pole's oscillation and a spouting period greatly.

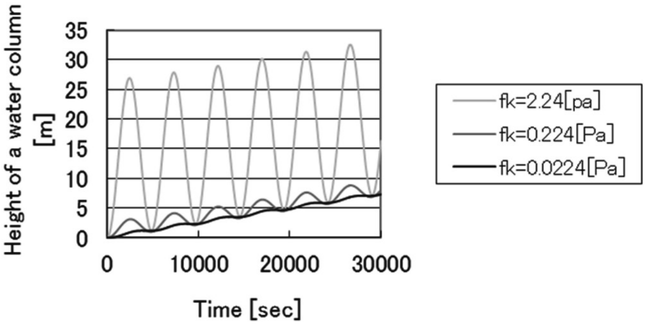

Fig. 5　Dependence of variation of height of a water pole (*x*) on pressure due to surface tension on the lower interface between water and gas (f_k).

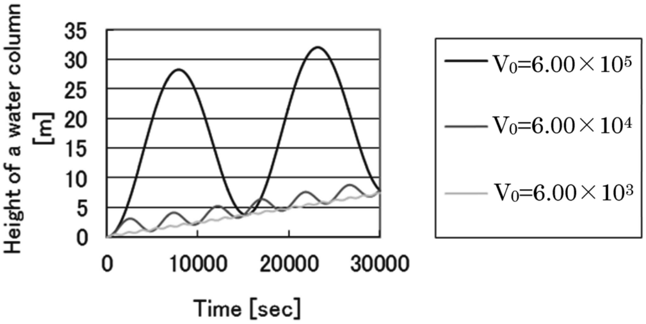

Fig. 6　Dependence of variation of height of a water pole (*x*) on volume of underground space (V_0 [m^3]).

3. 2 Results of numerical simulation of the improved dynamical model

I show the results of numerical simulation of the improved dynamical model of a geyser induced by the inflow of gas as follows.

The difference of variation of height of a water pole (x) from start to 8000 second after between in case of friction's existing (this improved model) and in case of no friction is shown in Fig. 7. From Fig. 7, we see that if there is friction, the amplitude of the water pole's oscillation becomes smaller as time passes. Then the same graph from 8000 second after to 16000 second after from the start is shown in Fig. 8. We cannot see downward movement of a water pole

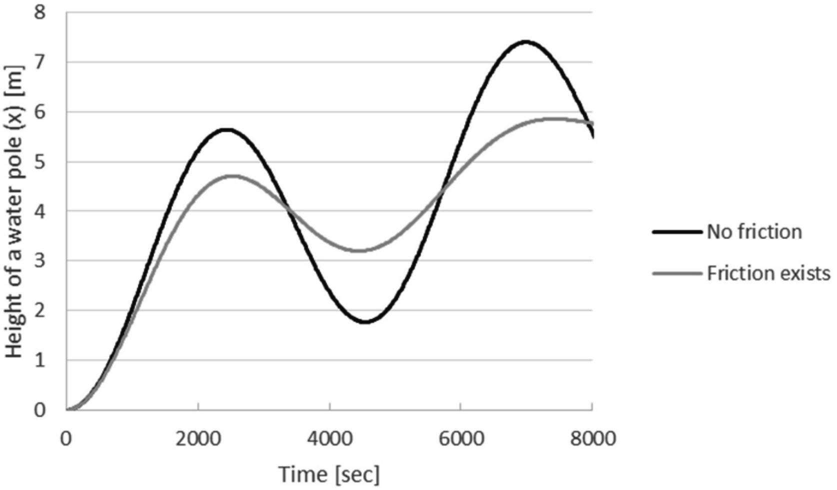

Fig. 7 Difference of variation of height of a water pole (x) from start to 8000 second after between in case of friction's existing and in case of no friction.

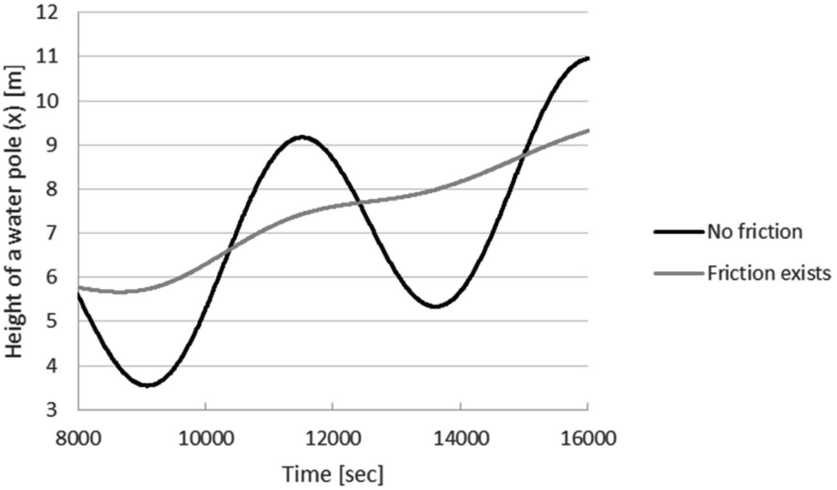

Fig. 8 Difference of variation of height of a water pole (x) from 8000 second after to 1600 second after since start between in case of friction's existing and in case of no friction.

almost after 9000 second after from the start if there is friction. This variation of height of a water pole resembles spouting dynamics of a geyser that involves no water pole's oscillation.

3.3 Application of the dynamical model to a real geyser induced by gas inflow

We can estimate the values of underground parameters through comparing spouting dynamics of real geyser induced by gas inflow with that of numerical simulation of the dynamical model. Concretely we select values of underground parameters as spouting dynamics of numerical simulation of the dynamical model fits that of real geyser induced by gas inflow, that is, as a spouting period, amplitude of a water pole's oscillation and so on of the numerical simulation fit those of real geyser when we do numerical simulation of the dynamical model. It is thought that the selected values of underground parameters are close to those of the real geyser induced by gas inflow. In fact, values of underground parameters of a few real geysers were already estimated due to this method by me (Kagami, 2006 ; Kagami, 2012a).

4. Conclusion

I constructed a basic dynamical model of a geyser induced by gas inflow through detailed observation of the indoor model experiments. And I clarified dependence of spouting dynamics on various underground parameters (volume of the underground space, length (height) of a water pole and so on) through numerical simulation of the dynamical model.

And I improved the dynamical model, that is, I took effects of friction between the walls of the spouting pipe and water into account. And I clarified how spouting height was damped by friction between the walls of the spouting pipe and water through numerical simulation of the improved dynamical model.

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A part of this paper, namely, derivation of a basic dynamical model of a geyser induced by gas inflow and an improved dynamical model in which effects of friction working between the wall's surface along the edge of the paths of water and water are taken into account was already published in "Proceedings of The 38th Conference of Sciete Internationale des Techniques Hydrothermales and The 56th Annual Meeting of the Balneological Society of Japan" (2003) and "Data Science Journal" (2010), though results of numerical simulations of the basic and improved dynamical model were not published in the proceedings and the journal. And I summarized a part of my doctoral thesis on this paper after making the revision.

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